9.3

Probability

If E is an event in a finite, nonempty sample space S of equally likely outcomes, then the **probability** of the event E is

$$P(E) = \frac{\text{the number of outcomes in } E}{\text{the number of outcomes in } S}$$

Example:

Find the probability of rolling a sum divisible by 3 on a single roll of two fair dice.

Solution: The event E consists of outcomes {3, 6, 9, 12}. To get the probability of E we add up the probabilities of the outcomes in E.

$$P(3) + P(6) + P(9) + P(12) = \frac{2}{36}+\frac{5}{36}+\frac{4}{36}+\frac{1}{36}=\frac{12}{36}=\frac{1}{3}.$$

A **probability function** is a function **P** that assigns to each outcome in a sample space a unique real number, subject to the following conditions:

1. $0 \leq P(O) \leq 1$ for every outcome O.
2. The sum of the probabilities of all outcomes in S is 1
3. $P(\varnothing) = 0$

Probability of an Event (Outcomes not Equally Likely)

Let S be a finite, nonempty sample space in which every outcome has a probability assigned by a probability function P. If E is an event in S, the

probability of the event E is the sum of the probabilities of all the outcomes contained in E.

Testing a probability function:

Example: Is it possible to weight a standard 6-sided die in such a way that the probability of rolling each number n is exactly $\frac{1}{\left(n^2+1\right)}$?

Solution:

The probability distribution would look like this:

Outcome	Probability	
1	$\frac{1}{2}$	
2	$\frac{1}{5}$	This is not a valid probability function,
3	$\frac{1}{10}$	because the sum of the probabilities
4	$\frac{1}{17}$	does not equal 1.
5	$\frac{1}{26}$	
6	$\frac{1}{37}$	

Example:

Jessie opens a box of a dozen chocolate crèmes and generously offers two of them to Jackie. Jackie likes the cherry crèmes the best, but all the chocolates look alike on the outside. If four of the twelve crèmes are cherry, what is the probability that both of Jackie's picks turn out to be cherry?

Solution: Two chocolates will be chosen, without regard to order, from a box of 12. There are ${}_{12}C_2 = 66$ outcomes all equally likely.

The event E consists of all possible pairs of 2 cherry crèmes that can be chosen, without regard to order, from 4 cherry crèmes available. There are ${}_4C_2 = 6$ ways to form such pairs.

Therefore, P(E) = $\frac{6}{66} = \frac{1}{11}$.

Multiplication Principle of Probability:

Suppose an event A has probability p_1 and an event B has probability p_2 under the assumption that A occurs. Then the probability that both A and B occur is p_1p_2.

P(A and B) = P(A) • P(B)

Another way to look at the Chocolate problem!

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cherry, what is the probability that both of Jackie's picks turn out to be cherry?

Solution:

As far as Jackie is concerned there are two types of chocolates cherry (C) and not cherry (N). When choosing two chocolates, there are four possible outcomes: CN, CC, NC, NN. We need to determine the probability of the outcome CC.

The probability of picking a cherry on the first draw is $\frac{4}{12}$. The probability of picking a cherry on the second draw, assuming cherry was picked on the first, is $\frac{3}{11}$. Using the Multiplication Principle the probability of drawing a cherry crème on both draws is: $\frac{4}{12} \cdot \frac{3}{11} = \frac{1}{11}$.

Conditional Probability

If the event B depends upon the event A, then P(B/A) = $\frac{P(A \text{ and } B)}{P(A)}$.

Example:

Two identical cookie jars are on a counter. Jar A contains 2 chocolate chip and 2 peanut butter cookies, while Jar B contains 1 chocolate chip cookie. Suppose we have drawn a cookie at random from one of the jars. Given that it is chocolate chip, what is the probability that it came from Jar A?

Solution:

P(jar A/chocolate chip) = $\dfrac{P(\textit{jar A and chocolate chip})}{P(\textit{chocolate chip})}$

$$= \frac{(1/2)(2/4)}{(1/2)(1/2)+(1/2)(1/1)} = \frac{(1/4)}{(3/4)} = \frac{1}{3}$$